PH16212, Homework 8

Deadline Dec. 30, 2019

1. (Differential equation for one-loop massive box) Consider one-loop one-external mass box with propagators,

$$D_1 = l^2, \quad D_2 = (l-p_1)^2, \quad D_3 = (l-p_1-p_2)^2, \quad D_4 = (l+p_4)^2 \tag{1}$$

with the kinematics,

$$p_1^2 = p_2^2 = p_4^2 = 0, \quad p_3^2 = a \tag{2}$$

$$p_1 \cdot p_2 = s/2, \quad p_1 \cdot p_4 = t/2, \quad p_2 \cdot p_4 = \frac{a-s-t}{2} \tag{3}$$

We select the master integrals

$$I = \{G[1,1,1,1], G[1,0,1,0], G[0,1,0,1], G[0,0,1,1]\} \tag{4}$$

- Use LiteRed, FIRE or Kira to derive the differential equations

  $$\frac{\partial}{\partial s} I = A_s I, \quad \frac{\partial}{\partial t} I = A_t I, \quad \frac{\partial}{\partial a} I = A_a I \tag{5}$$

  Write down the three matrices A_s, A_t and A_a.

- Explicitly check the integrability condition,

  $$\frac{\partial}{\partial t} A_s - \frac{\partial}{\partial s} A_t - [A_t, A_s] = 0 \tag{6}$$

- Compute the Euler relation

  $$s A_s + t A_t + a A_a \,. \tag{7}$$

2. Again for the one-loop one-massive box in the same configuration as the previous problem, find the UT integral basis,

$$\tilde{I} = T I \,. \tag{8}$$

Hint: for the box diagram, use the maximal cut computation to determine the overall coefficients. For the bubble diagram, you may use the double propagator trick.

Write down the new differential equations that

$$\frac{\partial}{\partial s} \tilde{I} = \tilde{A}_s \tilde{I}, \quad \frac{\partial}{\partial t} \tilde{I} = \tilde{A}_t \tilde{I}, \quad \frac{\partial}{\partial a} \tilde{I} = \tilde{A}_a \tilde{I} \tag{9}$$

Check that $\tilde{A}_s$, $\tilde{A}_t$ and $\tilde{A}_a$ are proportional to ϵ. ($D = 4 - 2\epsilon$). Check that

$$[\tilde{A}_s, \tilde{A}_t] = 0, \quad \partial_s \tilde{A}_t = \partial_t \tilde{A}_s = 0 \tag{10}$$